Magneto Hydrodynamic Nanofluid Flow over Convectively Heated Plate on Radially Stretching Sheet Embedded on Porous Media

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Cite: Alai, K.A., Wafula, M. & Njau L. (2023). Magneto Hydrodynamic Nanofluid Flow over Convectively Heated Plate on Radially Stretching Sheet Embedded on Porous Media. *The University Journal, 5(3),* 213-223.

Abstract

Fluid flow through this media is a key component in a wide range of activities like the generation of fluids from subsurface reservoirs and subterranean water resource restoration. Consequently, the study's goal is to investigate the motion of Magneto Hydro Dynamics (MHD) nanofluids across a convectively heated plate superimposed on a radially expanding sheet embedded in a porous media. The model is formulated and non dimensionalized using similarity variables. By employing the shooting technique to transform the boundary conditions and, the R-K scheme in MATLAB bvp4c, the system of ODEs is solved. The results obtained are displayed in graphs and others in tables. The results indicate that with increasing porosity, magnetism, and surface rotation, the flow primary velocity decreases while the temperature profile surges. The findings from this study will provide beneficial theoretical insight into what parameters should be varied for maximum profit in several sectors like the power engineering sector, aerodynamic combination, drug recovery systems, and water solar heating systems.

Keywords: MHD, porous media, expanding sheet

Introduction

A fluid is a substance that is easily deformed by external pressure and lacks a clearly defined shape. A nanofluid is a fluid that contains nanoparticles, or particles that are smaller than a nanometre. Colloidal suspensions of nanoparticles in the base fluid produce these fluids. Nanoparticles are typically made up of elements such as carbides, carbon nanotubes, metals, and oxides. As an illustration, the baseline fluids include ethylene, water, glycol, oil, and a variety of others. Numerous significant and practical characteristics of nanofluid, like their rate of stretching and rate of increase in heat transfer, have been found. Nanofluids have comprehensive potential applications, including those that are in the fields of aerodynamics and power engineering; heat exchangers; the cooling of transformers; chemical separation devices; solar water heating; micro pumps; and drug recovery systems. Zami and Ishak (2016) examined numerically a stretched medium through the study of steady fluid flow, a similar study was conducted by Alzahrani et al. (2019), Hayat et al. (2016), and Wakif (2020). Hall effects on MHD rotational viscoelastic fluid flow via a porous medium over an infinitely oscillating porous sheet were investigated by Krishna and Kamboji (2018). They agreed that the top wall shear stress remains positive during the whole oscillation cycle. Axial velocity along the channel increases with increasing penetrability of the porous medium, as shown in a study by Reddy et al. (2018). To improve the velocity profiles of both carbon nanotubes, Bilal and Ramzan (2019) found that an increased estimate of the Hall current parameter was beneficial. This came about as a result of their discussion regarding increasing the Hall current parameter estimate on both carbon nanotube velocity profiles. According to Rani et al. (2020), they studied the Hall and Iron slip on MHD nanofluid effect on flow over a vertical semi-infinite plate embedded in a porous media. They concluded that an unsteady perimeter has little effect on Casson nanofluid velocity distribution. A Hall current parameter causes a moderate X-direction velocity increase and a massive transversal velocity increase. The Casson parameter improves the temperature distribution, as discovered by Shah et al.(2020), in their study, when viscous and Joule dissipation effects are not present. They reached this conclusion after investigating viscous and Joule dissipation effects on temperature. Casson parameter decreases temperature when viscous and Joule dissipations are present. Nanofluid flow with radiation effects inside a stretched surface was studied by Shoaib et al. (2020), and they found that the rotation parameter for the velocity profile fluctuated. Scientists noticed that the parameter's rotation orientation had changed. Alhussain and Tassadiq (2021) used the stretching sheet to study viscosity fluctuations in a blood-based Casson nanofluid. Adding a perpendicular magnetic field to the flow field helped. Researchers found a link between nanomaterial content in base fluid and thermal expansion rate and specific heat capacity. In their study on the effects of the Hall on unsteady Couette flow and layer approximations boundary conditions, Kanch and Jana (2001) found that impulsive changes in pressure gradient impact on rotating magnetic field MHD Couette flow (Ghosh, 2002). He showed that the small-time solution's velocity components converge faster. Chauhan and Rastogi (2009) studied heat transfer and the Hall effect on MHD flow in a half-filled porous media channel. Later Chauhan and Agrawal (2010) studied Hall current and heat transfer in a porous, spinning channel. Seth et al. (2012) examined the rotation and Hall effect on MHD unstable Couette flow in an inclined magnetic field. In most cases, rotation increases the secondary flow while slowing the primary flow as revealed by Sarojini et al. (2012) investigation. Sarojini et al. (2012) studied the influence of an angled magnetic field of two stress fluids in a porous medium channel. In unsteady MHD three-dimensional flows of two stress fluids in parallel plates across a porous media, Raju et al. (2013) discovered that shear stress T_x induced by the main flow declines for impulsive change and cosine oscillations with an increase in magnetic variable. Krishna and Ahmed (2014) examined the effect of Hall on unstable MHD Maxwell fluid flow through a porous media. When the magnetic parameter is low, shear stress increases, while both cosine and sine pressure gradient oscillations decrease as frequency increases. Krishna and Prakash (2015) examined the impact of the Hall Effect on hydromagnetic unstable flow in a whirling parallel sheet with a porous bed. When a pressure gradient and hall current were present, rotational and Lorentz forces affected the speed profile. Flows over rotating surfaces have been thoroughly studied under several conditions. A study by Oke (2022) focuses on the MHD flow over a convectively heated plate on a radially stretching sheet through a porous medium.

Governing equations

This study considers the flow in a porous medium with the surface rotating at an angular velocity of **Ω**. The surface is a disk also stretching radially as shown in Figure 1.

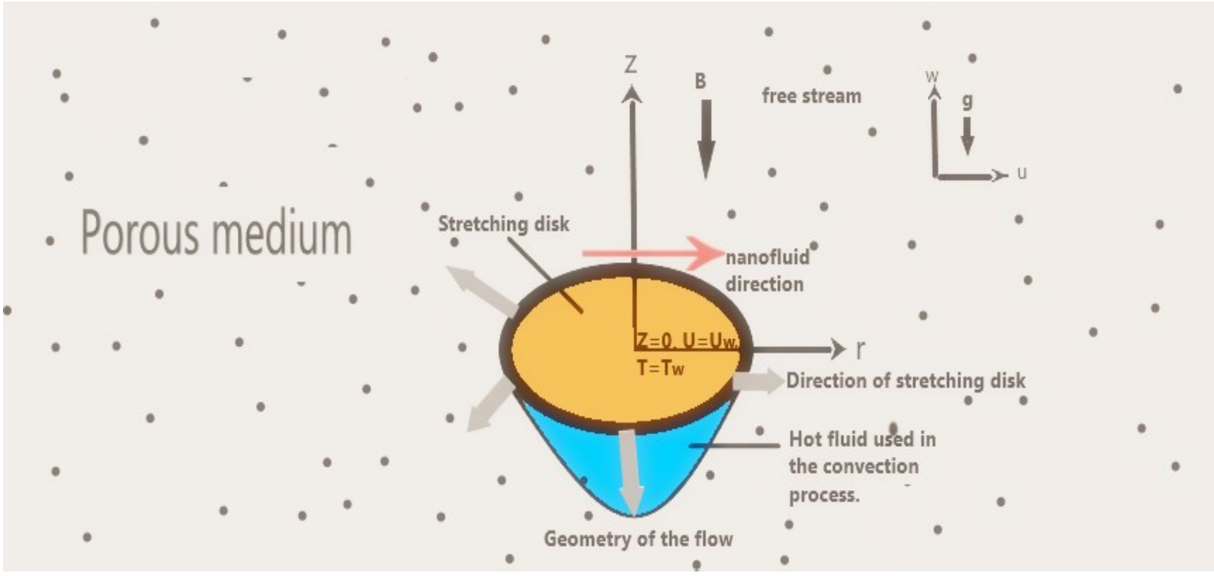

Figure1. Flow geometry

The nanofluid flow contains copper in one and alumina nanoparticles in another. The nanoparticles are preferred because they are scientifically proven to be efficient in heat transfer with minimal drawbacks in the suspension in the base fluid process. The region of interest is *z* >0 with the flow moving at a velocity of *u* on the *r*-axis and *w* on the *z*-axis. The nanofluid flow is experiencing a magnetic field of strength B. The disk is heated convectively by a hot fluid at the bottom. By considering our flow assumptions above, our flow model of the continuity equation, momentum equation and energy equation is given as;

$$\frac{1}{r}\frac{\partial(ru)}{\partial r} + \frac{\partial w}{\partial z} = 0. \qquad (1)$$

$$u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} = \frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 u}{\partial z^2} - \frac{1}{k}u\right) + 2\Omega u - \frac{\sigma_{nf}}{\rho_{nf}}B^2 u \qquad (2)$$

$$u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z} = \frac{1}{(\rho C_p)_{nf}}\left(k_{nf}\frac{\partial^2 T}{\partial z^2} - \frac{\mu_{nf}}{r^2}\left(\frac{\partial u}{\partial z}\right)^2 - Q_0\,(T - T_\infty)\right) \qquad (3)$$

The boundary conditions given as

$$Z=0: u = U_0 r,\ w = 0,\ z = 0: u = U_0 r, w = 0, -k_f\left(\frac{\partial T}{\partial Z}\right) = h_f\,(T_W - T) \qquad (4)$$

$$z \to \infty: u \to 0, T \to T_\infty. \qquad (5)$$

Methodology

By using the similarity variables;

$$\eta = \eta(z) = z\sqrt{\frac{U_0}{v_f}},$$

$$u(r,z) = U_o r \frac{df}{d\eta},$$

$$w(r,z) = -2\sqrt{(U_0 v_f)}f,$$

$$T(r,z) = T_\infty + (T_w - T_\infty)\Theta.$$

The system of governing equations (Eqs. (1) – (3) is transformed to the dimensionless form as

$$f''' - rf' - \frac{A_2}{A_1}(Kf' + Mf' + (f')^2 - 2ff'') = 0 \tag{6}$$

$$\Theta'' - Pr\left(\frac{A_1 PrEc}{B_1}(f'')^2 - \frac{B_2}{B_1}Q\Theta + 2\frac{B_2}{B_1}\Theta'\right) = 0 \tag{7}$$

Where $\frac{\mu_{nf}}{\rho_{nf}} = \frac{(1-\phi\ \ )^{-2.5}\mu_f}{\left(1-\phi+\phi\frac{\rho_{np}}{\rho_f}\right)\rho_f} = \frac{A_1}{A_2}v_f,$

And $A_1 = (1-\phi\ \ )^{-2.5}$

$$A_2 = 1 - \phi + \phi\frac{\rho_{np}}{\rho_f}.$$

With dimensionless boundary conditions as

$$\begin{cases} \eta = 0;\ f' = 0, B_i(\Theta - 1) = \Theta' \\ \quad \eta \to \infty: f' \to 0, \Theta \to 0. \end{cases} \tag{8}$$

Where the dimensionless parameters are prandtl number, Eckert number, Magnetic field parameter, Rotation parameter and volumetric rate of heat generation defined as

$$Pr = \frac{v_f}{\alpha_f},\ Ec = \frac{U_0^2}{(T_W - T_\infty)(C_p)_f}, M = \frac{\sigma_{nf}B^2}{U_0\,\rho_{nf}}, K = \frac{2\Omega}{U_0}, Q = \frac{Q_0}{(\rho C_P)_{nf}U_0}, \gamma = \frac{\boldsymbol{v_f}}{\boldsymbol{U_0 k}}, \alpha_f = \frac{k_f}{(\rho C_p)_f},$$

Eqs. (6) – (7) can be written as a system of first order ordinary differential equations by using the transformations

$$x_1 = f, x_2 = f', x_3 = f'', x_4 = \Theta, x_5 = \Theta'$$

Hence the system becomes:

$$\begin{cases} x_1' = x_2\,, \\ x_2' = x_3 \\ x_3' = \gamma x_2 + \left(\frac{A_1}{A_2} K x_2 + M x_2 + (x_2)^2 - 2x_1 x_3\right), \\ x_4' = x_5, \\ x_5' = Pr\left(\frac{A_1 Pr Ec}{B_1}(x_3)^2 - \frac{B_2}{B_1} Q x_4 + 2\frac{B_2}{B_1} x_5\right), \end{cases} \tag{9}$$

Numerical solution

Solving boundary value problem (BVP) system for the numerical solution is often daunting task. Shooting techniques (ShT) are usually employed to rewrite the BVP as an initial value problem (IVP) and this method is also adopted in this study. By adopting the ShT, a two-point BVP is transformed to a two-point IVP. To find the correct missing initial condition using the given boundary conditions, MATLAB shooting method allows initial estimates which is continuously refined by solving the system iteratively until the most accurate value is reached. At this point, the ODE system and the initial conditions obtained can be solved with our desired numerical method, which in our case is the Runge-Kuta of order 4 (R-K4) method. The fourth order RK method is employed as the main method of numerical solution in this study. This is because of the large region of stability and the scheme requires obtaining the slope at step *m* as follows;

$$S_1 = f(t_m, x_m),$$

$$S_2 = f\left(t_m + \frac{h}{2}, x_m + \frac{h}{2} S_1\right),$$

$$S_3 = f\left(t_m + \frac{h}{2}, x_m + \frac{h}{2} S_2\right),$$

$$S_4 = f(t_m + h, x_m + hS_3).$$

And next step is given by

$$x_{m+1} = x_m + \frac{h}{6}(S_1 + 2S_2 + 2S_3 + S_4),$$

$t_{m+1} = t_m + h.$

Where m = 0,1,2...

Results

In this chapter, the behavior of the velocity and temperature profiles under the influence of select parameter variation are analyzed and discussed. The graphs display the fluid behavior over different increasing parameters. The validation of results is also done and discussed. The graphs obtained from the simulations of the velocity and temperature profiles against increasing surface porosity, surface rotation and magnetic field strength are displayed. In the simulation, the parameters kept at constant values are:

$Pr = 1.6$, $Ec = 1$, $Q = 2$, $Bi = 0.8$, $\varphi = 0.15$.

The thermophysical properties of the nanoparticles are presented in the table (1).

Table 1. Thermophysical properties

	ρ	C_p	K	Σ	**Source**
Copper	8933	385	400	59.6×10^6	Khashi'ie*et al.* (2020)
Alumina	3970	765	40	35×10^6	Das *et al.* (2016)
base fluid	997. 1	417 9	0.613 0	5.5×10^{-6}	Das *et al.* (2016)

Figures 2 and 3 display the flow velocity and temperature behaviour with increasing surface porosity. The velocity reduces with increasing porosity while the temperature increases with increasing porosity. Figures 4 and 5 display the fluid velocity and temperature behaviour with increasing surface rotation. At increasing rotation, velocity reduces and temperature increases. Figures 6 and 7 display the fluid's velocity and temperature behaviour with increasing magnetic field strength. At increasing magnetic parameter, velocity reduces and temperature increases.

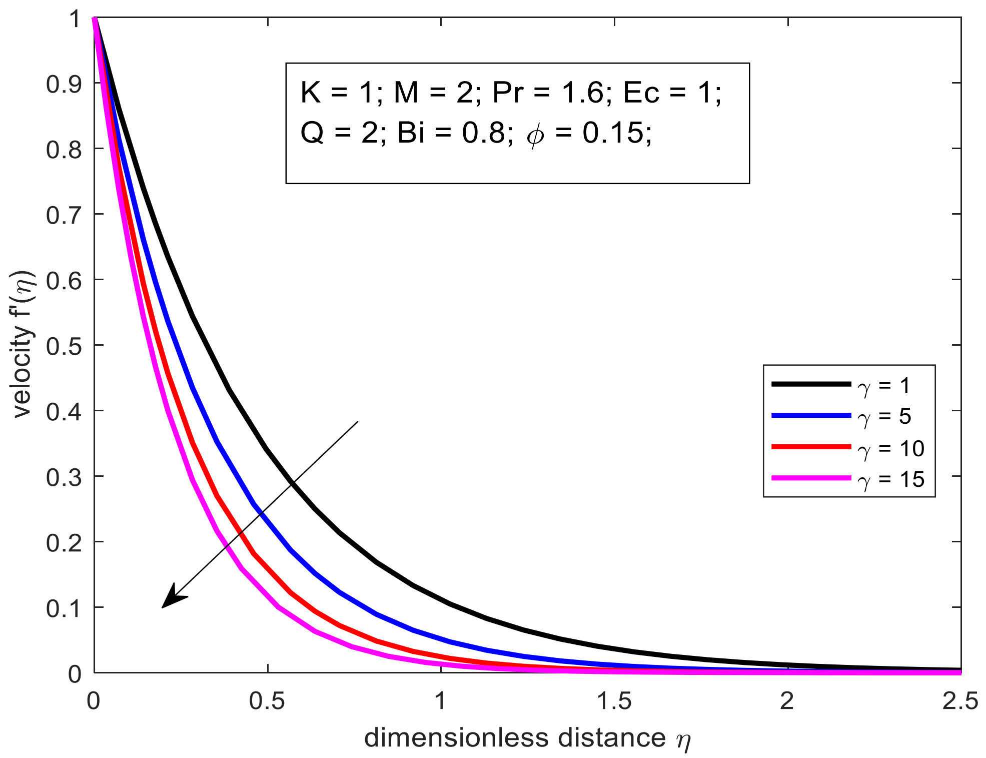

Figure 2. Response of velocity under increasing porosity parameter

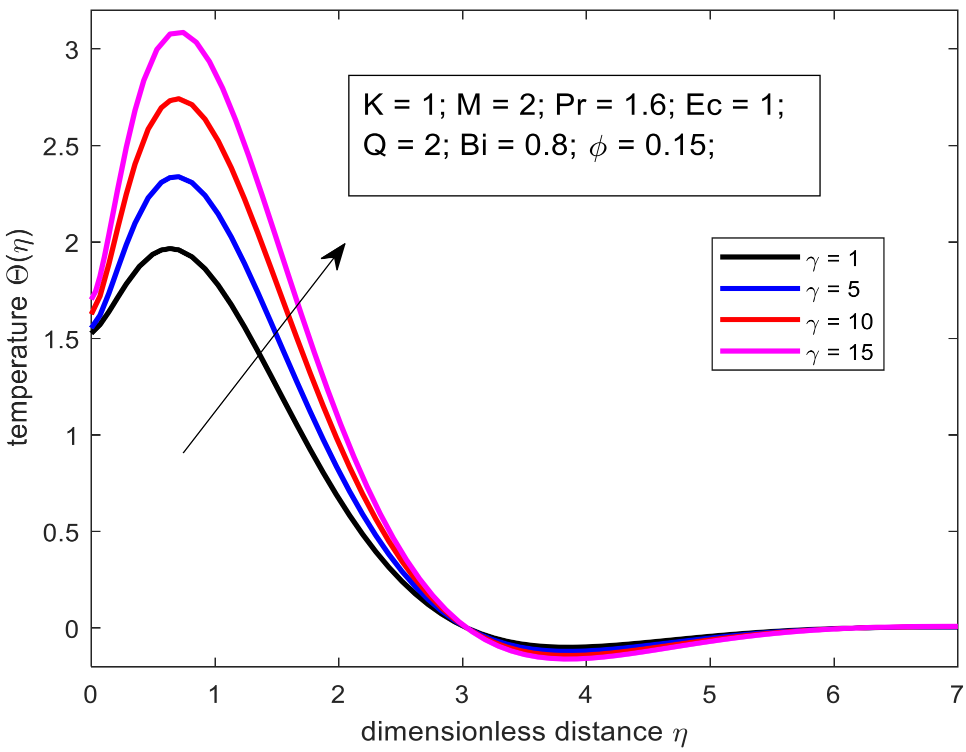

Figure 3. Response Temperature with dimensionless distance under increasing porosity parameter

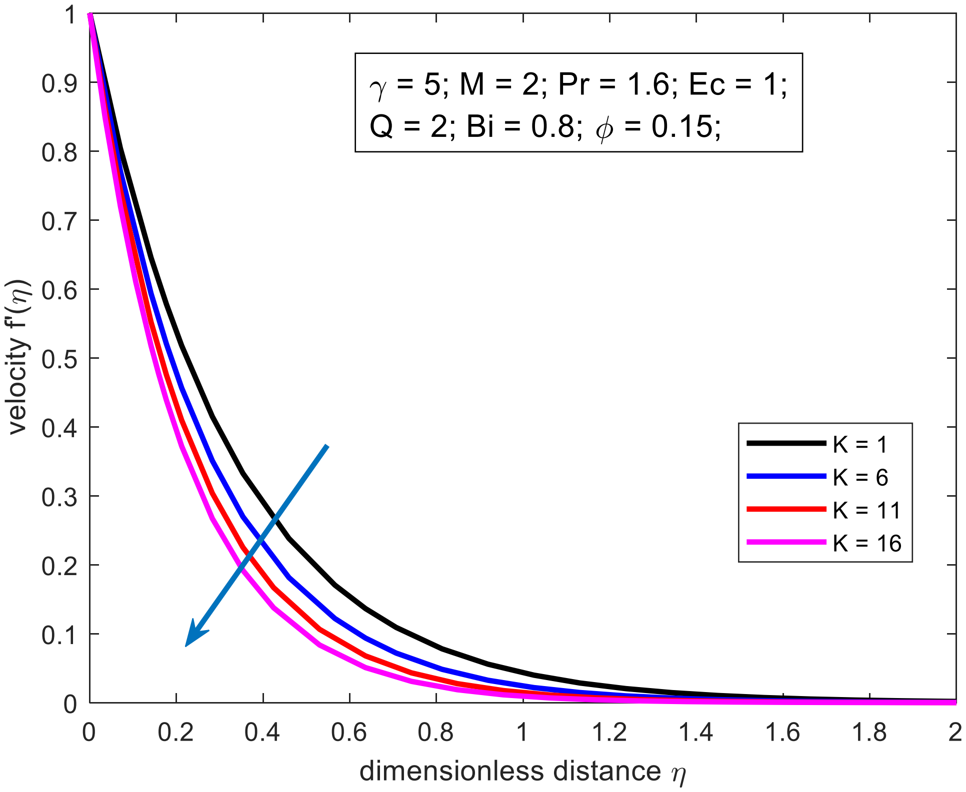

Figure 4. Response Velocity with dimensionless distance under increasing rotation

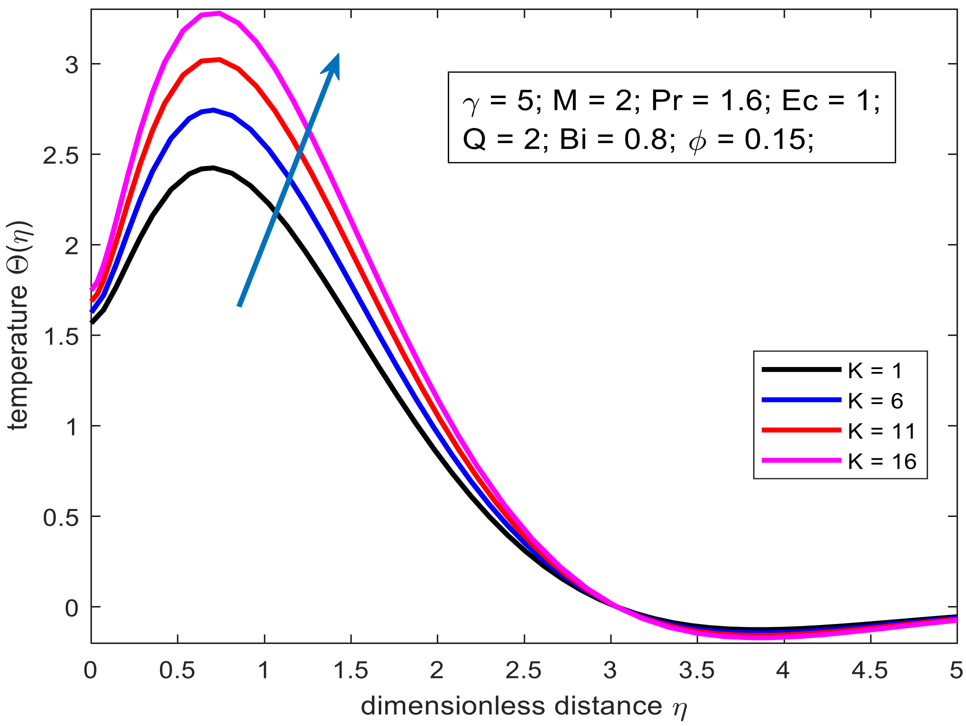

Figure 5. Response Temperature with dimensionless distance under increasing rotation

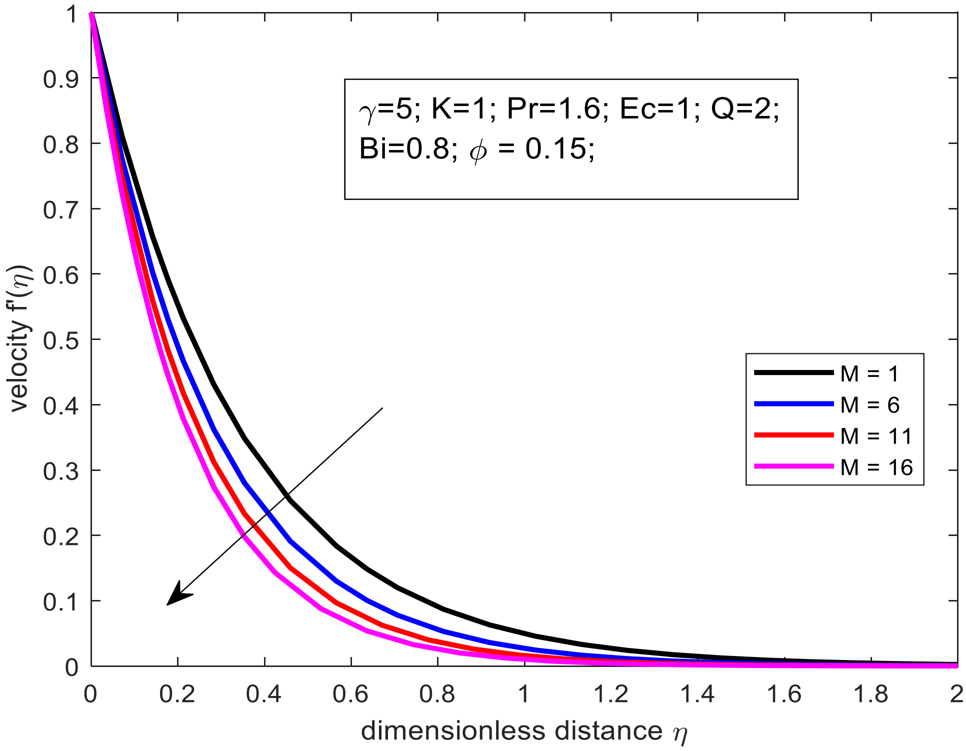

Figure 6. Response Velocity with dimensionless distance under increasing magnetic field

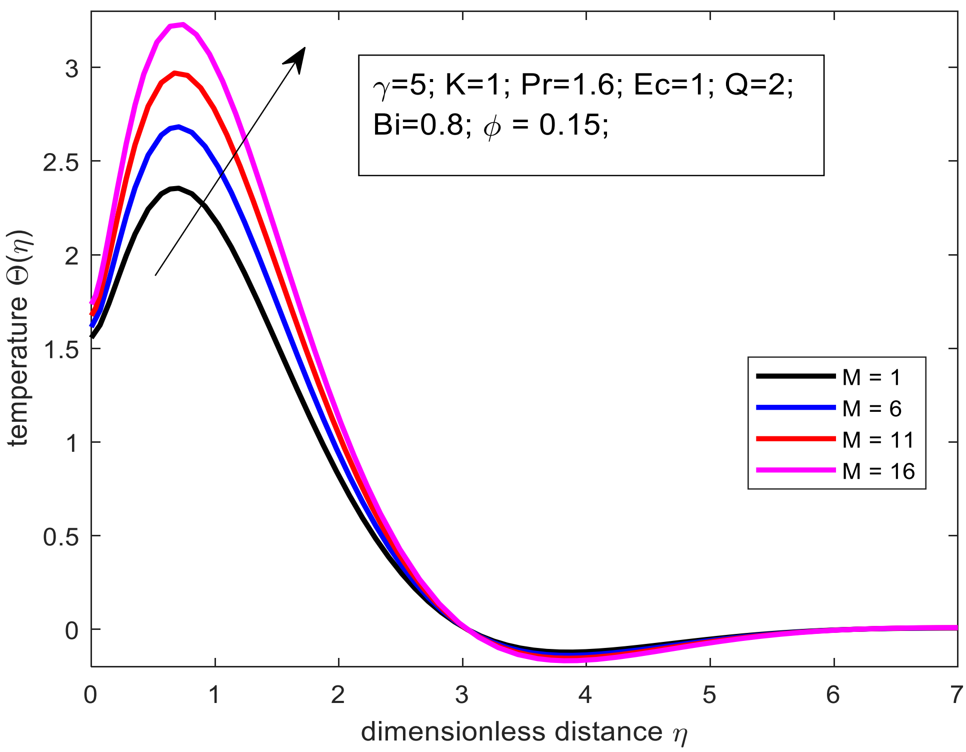

Figure 7. Response Temperature with dimensionless distance under increasing magnetic field

Discussion of Results

The fluid's primary velocity decreases with increase in the surface porosity as displayed in figure 1, this can be explained by the analogy that when you increase the surface porosity, this generates more viscous drag in the flow that in turn slows down the fluid flow rate. This results in the deceleration of the fluid velocity profile. The temperature of the fluid increases with increase in the surface porosity as observed in figure 2. This can be attributed to the fact that when porosity is increased, this restricts the fluid movement on the moving surface. This brings about viscous drag which leads to heat generation at the rotating surface. This heat together with the heat from the plate results in the continual heat generation as the porosity increases. The velocity profile declines with surging surface rotation as illustrated in figure 3 due to the increase in the friction between the fluid flow and the surface of the plate resulting from the increasing surface rotation. When this friction increases, the drag on the flow is increased and consequently decreasing velocity of the flow with continual increase in the rotation of the flow surface. The temperature of the fluid increases with increase in the surface rotation as shown in figure 4. When the rotation of the surface is increased, frictional force is generated between the fluid flow and the surface of the plate. This frictional force results in heat production at the point of contact of the flowing fluid and the rotating surface. The heat that is produced increases with the increase in the surface rotation. The heat at the contact point combined with the heat from the convectively heated plate causes the surge in temperature of the flow with continual increase in the rotation of the flow surface. When magnetic field is continually increase in the flow, the velocity of the fluid declines as demonstrated in figure 5. This can be attributed to the Lorenz force created in the flow. Given that Lorenz force is a drag force, this force offers resistance to fluid flow thereby reducing the rate of fluid movement. Overall, this results in the fluid primary velocity decline. As illustrated in figure 6, increasing magnetic field simultaneously increases the temperature in the flow. This can be as a result of Lorenz force generation. When magnetic field is surged in a fluid flow, another force called Lorenz force is created. This force results in more heat energy production which brings about an increase in temperature in the flow. Since, when the heat generated is added to the fluid internal temperature, there is temperature increase.

Conclusion

The study investigates the motion of two-dimensional, steady, incompressible and viscous MHD nanofluids across a convectively heated plate superimposed on a radially expanding sheet embedded in a porous medium. The governing equations 1 to 3 are formulated and non dimensionalised to equations 6 to 7 using similarity variables. By employing the shooting technique to transform the boundary conditions and, R-K4 scheme in MATLAB bvp4c, the system of ODEs are numerically solved. The results obtained are displayed in graphs and tables. The behaviour of the fluid velocity and temperature against various parameters are analysed and discussed. From the simulated graphs, the following observations are made;

a) Velocity decreases with increasing porosity, surface rotation and magnetic field strength.

b) Fluid temperature increases with increasing magnetic field, surface rotation and surface porosity.

c) As a result, we can draw the following conclusions that;

d) For a required velocity increase, the values of magnetic field, surface rotation and porosity of the surface must be reduced.

e) Increase in temperature can be achieved by increasing the magnetic field, surface rotation and surface porosity.

f) Surface drag can be reduced by reduction in surface rotation.

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